2016-2017 MM2MS2 Exam Solutions

1.

(a)

Load P produces bending moment and torque.

Bending moment is given by:

$$M = PL_{AB} = 10 \times 0.1 = 1 \text{ Nm}$$

[2 marks]

and the Torque, T is given by:

$$T = PL_{BC} = 10 \times 0.15 = 1.5 \text{ Nm}$$

[2 marks]

Axial stress at A due to the bending moment:

$$\sigma_z = \frac{My}{I} = \frac{64 \times 1 \times (2 \times 10^{-3})}{\pi \times (4 \times 10^{-3})^4} = 159.3 \times 10^6 \text{ Pa}$$

$$\boldsymbol{\sigma_z = 159.3 \text{ MPa}}$$

[2 marks]

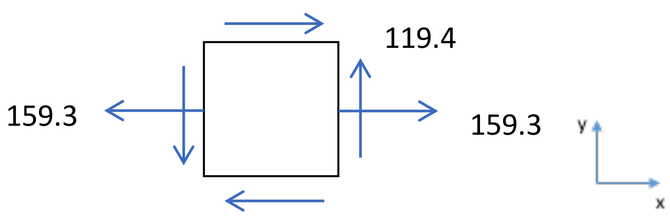
The torsional shear stress:

$$\tau = \frac{Tr}{J} = \frac{32 \times 1.5 \times (2 \times 10^{-3})}{\pi \times (4 \times 10^{-3})^4} = 119.4 \times 10^6 \text{Pa}$$

$$\boldsymbol{\tau = 119.4 \text{ MPa}}$$

[2 marks]

Plane stress state of an element from the wall:



[3 marks]

(b)

The centre of the Mohr's circle is:

$$C = \frac{(\sigma_x + \sigma_y)}{2} = \frac{159.3 + 0}{2} = 79.6 \text{ MPa}$$

[2 marks]

and the radius of the circle is:

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{159.3 - 0}{2}\right)^2 + 119.3^2} = 143.4 \text{ MPa}$$

[2 marks]

$$\sigma_{1,2} = C \pm R = 79.6 \pm 143.4$$

[1 mark]

The principal stresses are:

$$\boldsymbol{\sigma_1 = 223.1 \text{ MPa}}$$

[2 marks]

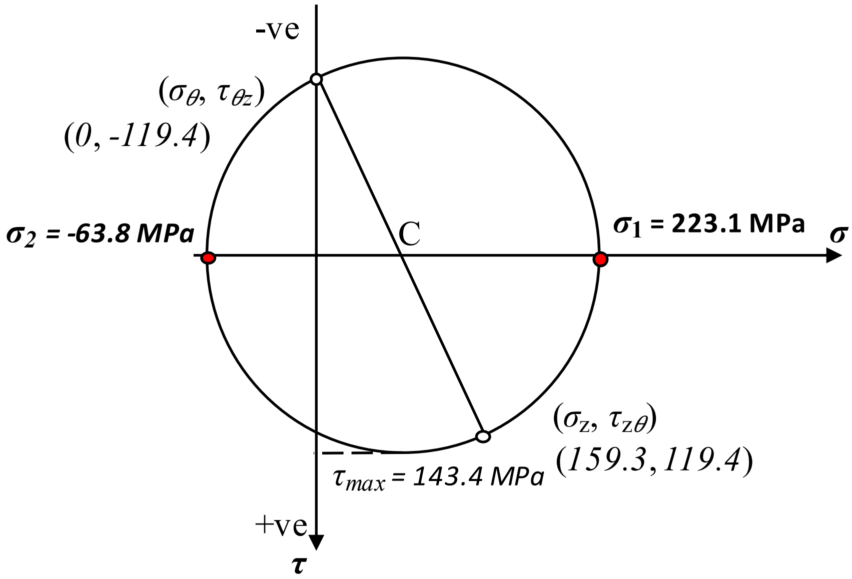
$$\boldsymbol{\sigma_2 = -63.8 \text{ MPa}}$$

[2 marks]

The maximum shear stress is:

$$\boldsymbol{\tau_{max} = R = 143.4 \text{ MPa}}$$

[1 mark]



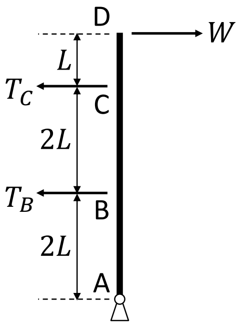
(Not to scale)

[4 marks]

2.

(a)

Free body diagram of the component:



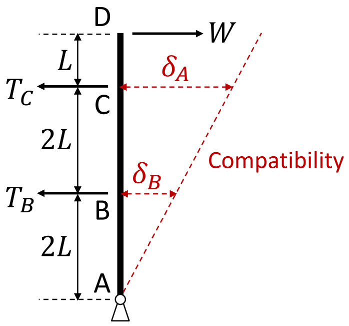
Taking moments about position A:

$$4T_C + 2T_B = 5W \qquad (1)$$

Compatibility:

$$\delta_C = 2\delta_B$$

as shown in the following diagram,



Hooke's law:

$$T_C = k_C\delta_C$$

and,

$$T_B = k_B\delta_B$$

Therefore,

$$\frac{T_C}{T_B} = \frac{k_C}{k_B} \times \frac{\delta_C}{\delta_B} = \frac{{D_C}^2}{{D_B}^2} \times \frac{2}{1} = \frac{800}{144}$$

$$\therefore T_B = 0.18T_C \qquad (2)$$

Simultaneously solving equations (1) and (2):

$$\boldsymbol{T_B = 0.20642W} \text{ (= reaction at E)}$$

and,

$$\boldsymbol{T_C = 1.4679W} \text{ (= reaction at F)}$$

(b)

$$\delta = \frac{F \times L}{A \times E} + \alpha \Delta T L$$

$$\therefore \frac{\delta_C}{\delta_B} = \frac{\frac{T_C}{A \times E} + \alpha \Delta T}{\frac{T_B}{A \times E} + \alpha \Delta T} = \frac{2}{1}$$

$$\therefore T_C A_B = 2T_B A_C = \alpha \Delta T E A_B A_C = \alpha \Delta T E \frac{\pi D_B}{4} \frac{\pi D_C}{4}$$

$$\therefore 76.7T_C = 346T_B = 1{,}338{,}000 \text{ N}$$

Simultaneously solving this and equation (1):

$$\boldsymbol{T_B = 0.2494W - 3480 \text{ N}} \qquad (3)$$

and,

$$\boldsymbol{T_C = 1.1253W + 1740 \text{ N}} \qquad (4)$$

(c)

$$T_{B_{allowable}} = \frac{F_{UTS_B}}{5} = \frac{102{,}000}{5} = 20{,}400 \text{ N} \qquad (5)$$

and,

$$T_{C_{allowable}} = \frac{F_{UTS_C}}{5} = \frac{231{,}000}{5} = 46{,}250 \text{ N} \qquad (6)$$

From (3) and substituting in (5):

$$W_B = \frac{T_B + 3{,}480}{0.2494} = \frac{24{,}000 + 3{,}480}{0.2494} = 95{,}750\ \mathrm{N}$$

Similarly, from (4) and substituting in (6):

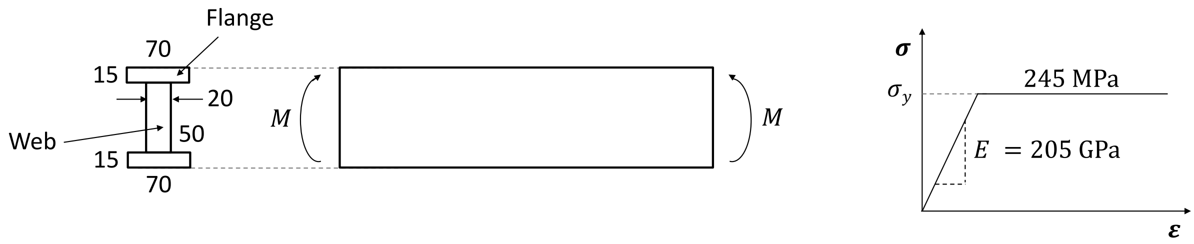
$$W_C = \frac{T_C - 1{,}740}{1.1253} = \frac{46{,}250 - 1{,}740}{1.1253} = 39{,}554\ \mathrm{N}$$

Therefore

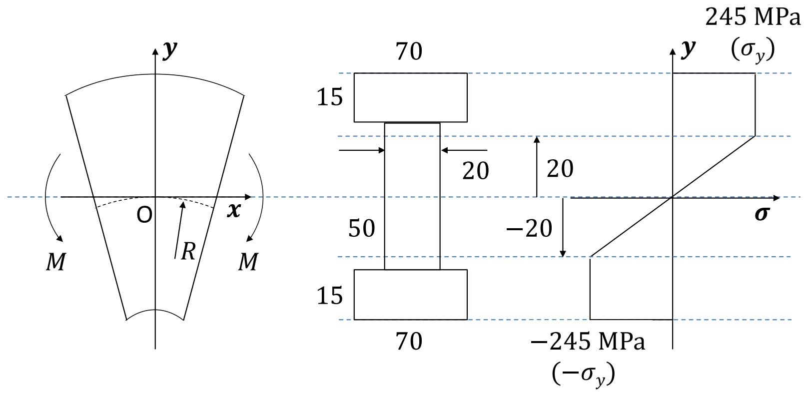
$$\boldsymbol{W_{max} = 39,554\ \mathrm{N}}$$

3.

(a)



Yielding will occur through whole of flange and 5 mm into each end of the web, therefore:



Variation of stress with y:

- For $20 < y < 40$, $\sigma = 245$ MPa
- For $-20 < y < 20$, $\sigma = \frac{245}{20} y$ MPa
- For $-40 < y < -20$, $\sigma = -245$ MPa

[2 marks]

Moment equilibrium:

(Balance the moments due to stresses in the elastic and plastic regions with the applied moment)

$$M = \int_A y\sigma dA = \int_y y\sigma b dy$$

[2 mark]

Due to the symmetry of the stress distribution and substituting in the elastic and plastic terms for σ, this can be rewritten as:

$$M = 2\left\{\int_0^{20} y\frac{245}{20}y(20)dy + \int_{20}^{25} y(245)(20)dy + \int_{25}^{40} y(245)(70)dy\right\}$$

$$= 2\left\{245\int_0^{20} y^2dy + 4{,}900\int_{20}^{25} ydy + 17{,}150\int_{25}^{40} ydy\right\} = 2\left\{245\left[\frac{y^3}{3}\right]_0^{20} + 4{,}900\left[\frac{y^2}{2}\right]_{20}^{25} + 17{,}150\left[\frac{y^2}{2}\right]_{25}^{40}\right\}$$

$$= 2\left\{245\left(\frac{20^3}{3}\right) + 4{,}900\left(\frac{25^2}{2} - \frac{20^2}{2}\right) + 17{,}150\left(\frac{40^2}{2} - \frac{25^2}{2}\right)\right\}$$

$$= 2\{653{,}833.33 + 551{,}250 + 8{,}360{,}625\}$$

$$\therefore \boldsymbol{M = 19{,}131{,}416.66\ \mathbf{Nmm} = 19.13\ \mathbf{kNm}}$$

[4 marks]

Compatibility:

$$\varepsilon = \frac{y}{R} \qquad (1)$$

[1 mark]

At $y = 20\ \text{mm}$, $\sigma = \sigma_y = 245\ \text{MPa}$ and since this point is within the elastic range:

$$\varepsilon = \frac{\sigma_y}{E} = \frac{245}{205{,}000} = 1.195 \times 10^{-3}$$

[1 mark]

Substituting this into (1) gives:

$$1.195 \times 10^{-3} = \frac{20}{R}$$

$$\therefore \boldsymbol{R = 16{,}736.4\ \mathbf{mm} = 16.74\ \mathbf{m}}$$

[3 marks]

(b)

$$I = \left(\frac{bd^3}{12}\right)_{outer} - \left(\frac{bd^3}{12}\right)_{gaps} = \frac{70 \times 80^3}{12} - 2\left(\frac{25 \times 50^3}{12}\right) = 2{,}986{,}666.67 - 520{,}833.33$$

$$= 2{,}465{,}833.34\ \text{mm}^4$$

[1 mark]

Unloading is assumed to be entirely elastic. Beam bending equation:

$$\frac{M}{I} = \frac{\sigma}{y}\left(= \frac{E}{R}\right)$$

$$\therefore \frac{\Delta M}{I} = \frac{\Delta \sigma}{y}$$

[2 marks]

Max change in stress ($\Delta\sigma$) will occur at $y = \frac{d}{2} = y_{max}$ $(= \pm 40 \text{ mm})$.

$$\therefore \Delta\sigma_{max}^{el} = \frac{\Delta M \times y_{max}}{I} = \frac{-M \times y_{max}}{I} = \frac{-19{,}131{,}416.66 \times \pm 40}{2{,}465{,}833.34}$$

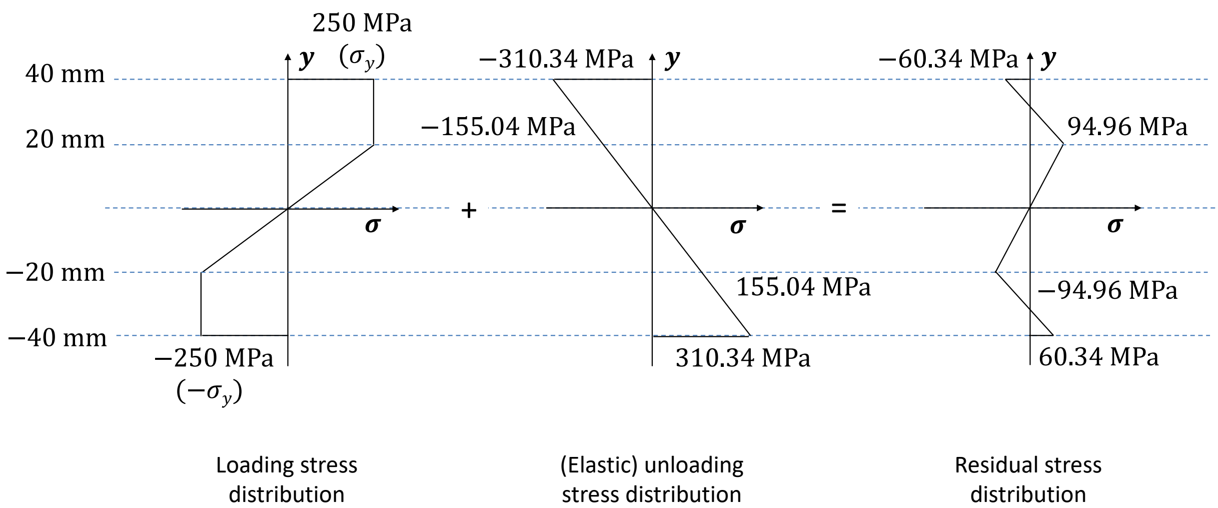
$$= \mp 310.34 \text{ MPa}$$

i.e.:

$$at\ y = 40 \text{ mm}, \therefore \Delta\sigma_{max}^{el} = -310.34 \text{ MPa}$$

$$and\ at\ y = -40 \text{ mm}, \therefore \Delta\sigma_{max}^{el} = 310.34 \text{ MPa}$$

[2 marks]



Interpolation of (elastic) unloading line:

At $y = 40$ mm, $\sigma = -310.34$ MPa
$y = m\sigma + c$
$\therefore 40 = \text{m} \times -310.34 + 0$
$\therefore \text{m} = -0.129$

At $y = 20$ mm, $20 = -0.129 \times \sigma$
$\therefore \sigma = -155.04$ MPa

[2 marks]

Residual stress is well below yield ($245\,\text{MPa}$), so reverse yielding does not occur. At $y = 20\,\text{mm}$, no plastic deformation occurs during loading and unloading.

$$\therefore \varepsilon_{residual} = \frac{\sigma_{residual}}{E} = \frac{94.96}{205{,}000} = 4.632 \times 10^{-4}$$

[1 mark]

Also,

$$\varepsilon = \frac{y}{R}$$

[1 mark]

$$\therefore 4.632 \times 10^{-4} = \frac{20}{R}$$

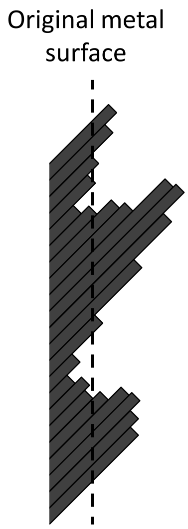
$$\therefore \boldsymbol{R = 43{,}177.89\ \text{mm} = 43.18\ \text{m}}$$

[3 marks]

4.

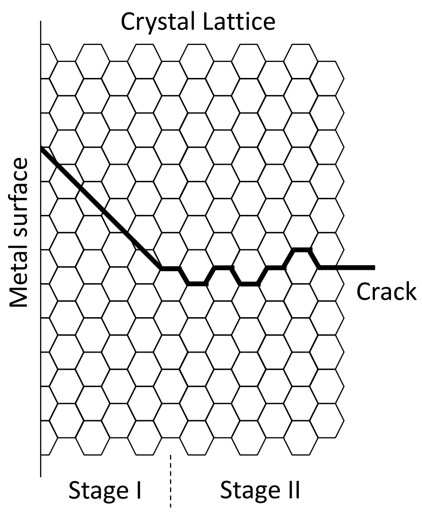
(a)

Crack Initiation and Stage I crack Growth: Pile up of dislocations causes slip bands, creating stress concentrations (features). This leads to shear stress controlled trans-granular cracking. This occurs on the plane on maximum shear (45° to the loading plane) as shown below.



[3 marks]

Stage II Crack Growth: Once the crack has reached a critical length, the stress state around the crack tip changes and cracks will propagate due to the maximum tensile stress (90° to the loading direction). This phase is usually intergranular.

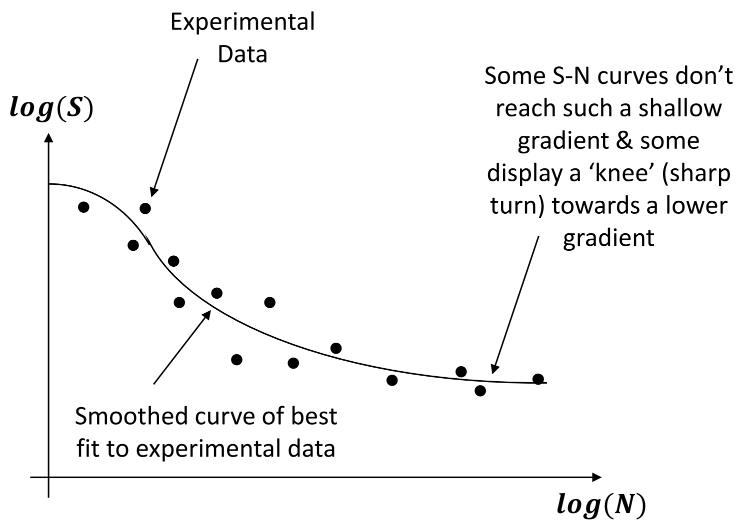


[4 marks]

Failure: Failure is at a critical crack length when the structure can no longer support the applied loads and it fails due to ductile tearing or cleavage (brittle) fracture.

[3 marks]

(b)

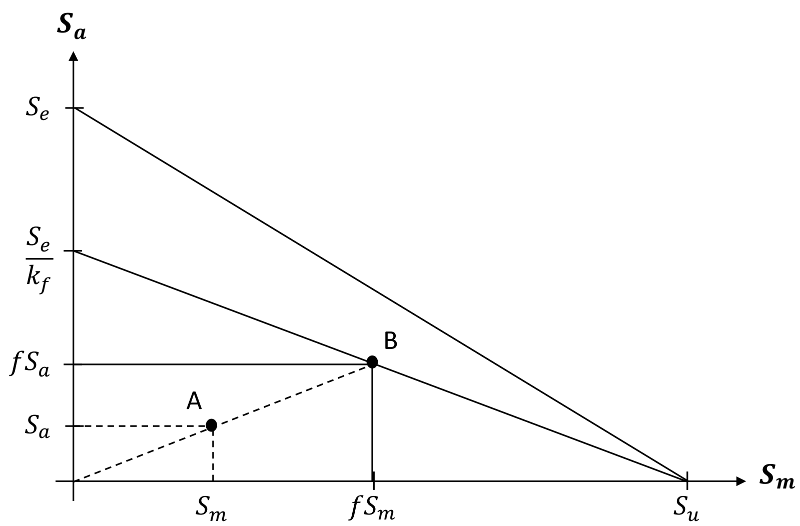


Fatigue strength is a hypothetical value of stress range at failure for exactly N cycles as obtained from an S-N curve.

The fatigue limit (sometimes called the endurance limit) is the limiting value of the median fatigue strength as N becomes very large, e.g. >10^8 cycles.

[5 marks]

(c)



[4 marks]

From similar triangles:

$$\frac{S_e}{k_f S_u} = \frac{fS_a}{S_u - fS_m}$$

$$\therefore S_a = \frac{S_e(S_u - fS_m)}{fk_fS_u} = \frac{100(300 - 1.1 \times 65)}{1.1 \times 1.75 \times 300} = \mathbf{39.57\ MPa}$$

[6 marks]

5.

(a)

$$A = 0.02 \times 0.02 = 4 \times 10^{-4}\ \mathrm{m}^2$$

$$L = 0.8\ \mathrm{m}$$

$$E = 200 \times 10^{9}\ \mathrm{Pa}$$

$$\frac{AE}{L} = \frac{4 \times 10^{-4}\ 200 \times 10^{9}}{0.8} = 100 \times 10^{6}\ \mathrm{N/m}$$

[2 marks]

The stiffness matrix of a truss element is:

$$[K_e] = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

Element 1, angle = 75°, $\cos\theta = 0.966$, $\sin\theta = 0.259$:

$$[K_{e1}] = (100 \times 10^{6})\begin{bmatrix} 0.067 & 0.250 & -0.067 & -0.250 \\ 0.250 & 0.933 & -0.250 & -0.933 \\ -0.067 & -0.250 & 0.067 & 0.250 \\ -0.250 & -0.933 & 0.250 & 0.933 \end{bmatrix}$$

[3 marks]

Element 2, angle = 225°, $\cos\theta = -0.7071$, $\sin\theta = -0.7071$:

$$[K_{e2}] = (100 \times 10^{6})\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

[3 marks]

Overall stiffness matrix for structure

$$[\boldsymbol{K}] = (\mathbf{100 \times 10^{6}})\begin{bmatrix} \mathbf{0.067} & \mathbf{0.250} & \mathbf{-0.067} & \mathbf{-0.250} & \mathbf{0} & \mathbf{0} \\ \mathbf{0.250} & \mathbf{0.933} & \mathbf{-0.250} & \mathbf{-0.933} & \mathbf{0} & \mathbf{0} \\ \mathbf{-0.067} & \mathbf{-0.250} & \mathbf{0.567} & \mathbf{0.750} & \mathbf{-0.500} & \mathbf{-0.500} \\ \mathbf{-0.250} & \mathbf{-0.250} & \mathbf{0.750} & \mathbf{1.433} & \mathbf{-0.500} & \mathbf{-0.500} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.500} & \mathbf{-0.500} & \mathbf{0.500} & \mathbf{0.500} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.500} & \mathbf{-0.500} & \mathbf{0.500} & \mathbf{0.500} \end{bmatrix}$$

[4 marks]

(b)

Horizontal and vertical components of force at B:

$$F_{B_H}\,(F3) = 25\cos 165 = -24.14\ \text{kN}$$

$$F_{B_V}(F4) = 25\sin 165 = 6.47\ \text{kN}$$

[2 marks]

Overall equations:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = (100 \times 10^6) \begin{bmatrix} 0.067 & 0.250 & -0.067 & -0.250 & 0 & 0 \\ 0.250 & 0.933 & -0.250 & -0.933 & 0 & 0 \\ -0.067 & -0.250 & 0.567 & 0.750 & -0.500 & -0.500 \\ -0.250 & -0.250 & 0.750 & 1.433 & -0.500 & -0.500 \\ 0 & 0 & -0.500 & -0.500 & 0.500 & 0.500 \\ 0 & 0 & -0.500 & -0.500 & 0.500 & 0.500 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

Applying BCs, $u_1 = u_2 = u_5 = u_6 = 0$, reduces the problem to:

$$\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = (100 \times 10^6) \begin{bmatrix} 0.567 & 0.750 \\ 0.750 & 1.433 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

Applying forces

$$\begin{bmatrix} -24140 \\ 6470 \end{bmatrix} = (100 \times 10^6) \begin{bmatrix} 0.567 & 0.750 \\ 0.750 & 1.433 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

[1 mark]

Therefore:

$$-24140 = (5.67 \times 10^7)u_3 + (7.5 \times 10^7)u_4 \qquad (1)$$

$$6470 = (7.5 \times 10^7)u_3 + (1.433 \times 10^8)u_4 \qquad (2)$$

From (1):

$$u_3 = \frac{-24140 - (7.5 \times 10^7)\,u_4}{5.67 \times 10^7} = -4.257 \times 10^{-4} - 1.323u_4$$

Subs this into (2):

$$6470 = 7.5 \times 10^7(-4.257 \times 10^{-4} - 1.323u_4) + (1.433 \times 10^8)u4 = -3.193 \times 10^4 + (4.41 \times 10^7)u_4$$

$$\therefore u_4 = \frac{-3.193 \times 10^4 - 6470}{-4.41 \times 10^7} = 8.71 \times 10^{-4}\ \mathrm{m}$$

[1 mark]

Substituting this into (1):

$$u_3 = \frac{-24140 - 7.5 \times 10^7 \times 8.71 \times 10^{-4}}{5.67 \times 10^7} = -1.578 \times 10^{-3}\ \mathrm{m}$$

[1 mark]

From matrix equation:

$$F_1 = -(6.7 \times 10^6)u_3 - (25 \times 10^6)u_4 = \mathbf{-11202\ N}$$

[2 marks]

$$F_2 = -(25 \times 10^6)u_3 - (93.3 \times 10^6)u_4 = \mathbf{-41814\ N}$$

[2 marks]

$$F_5 = -(50 \times 10^6)(u_3 - u_4) = \mathbf{35350\ N}$$

[2 marks]

$$F_6 = \mathbf{35350\ N}$$

(equation identical to that for F_5)

[2 marks]

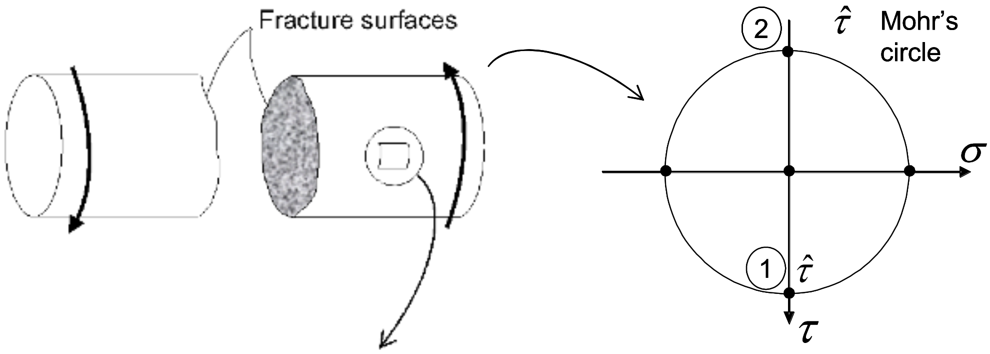
6.

(a)

Ductile:

Fracture plane is transverse to the axis of the specimen

[1 mark]



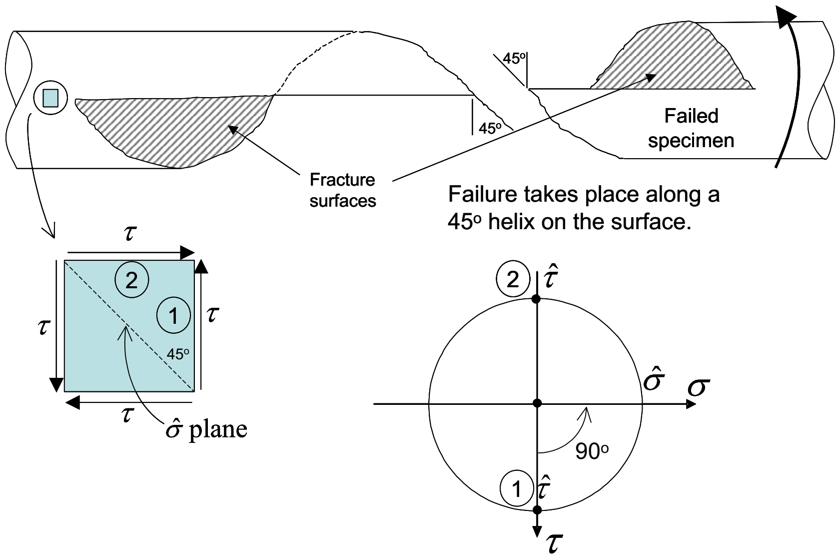
or: on the plane of maximum shear stress

[2 marks]

Brittle:

Helical failure

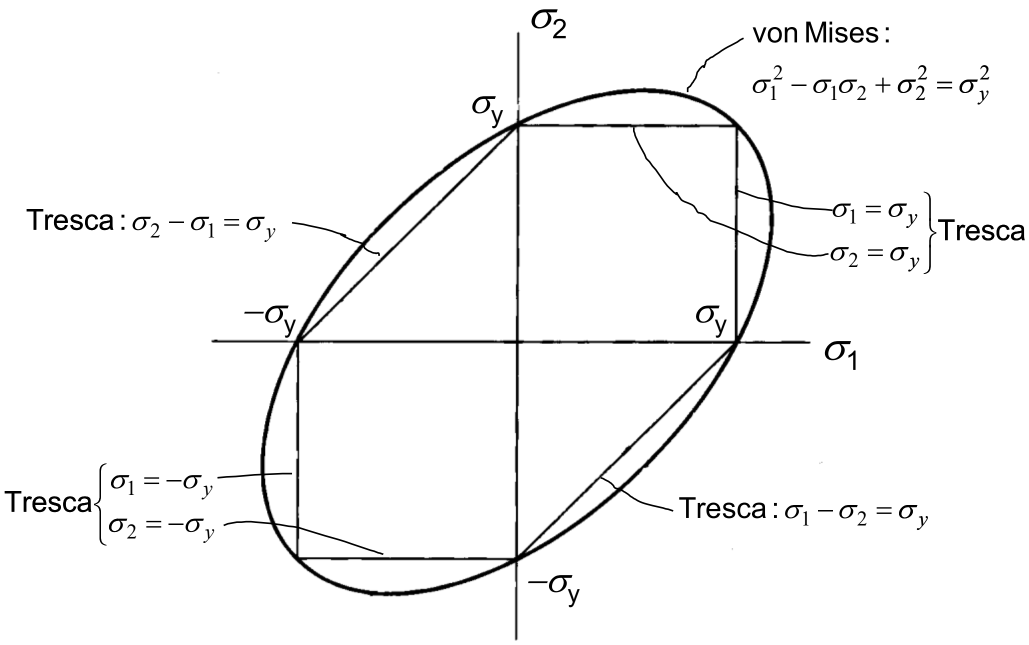
[1 mark]



or: on the plane of maximum principal stress

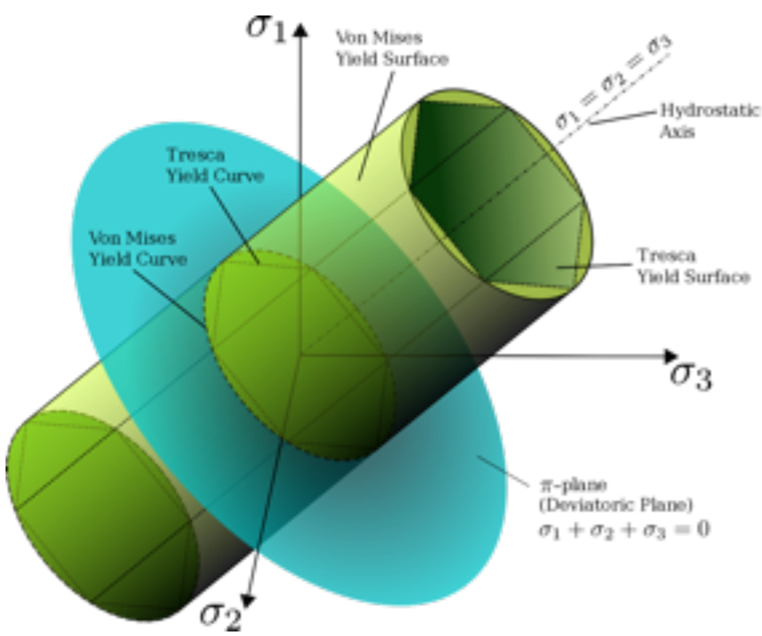
[2 marks]

(b)



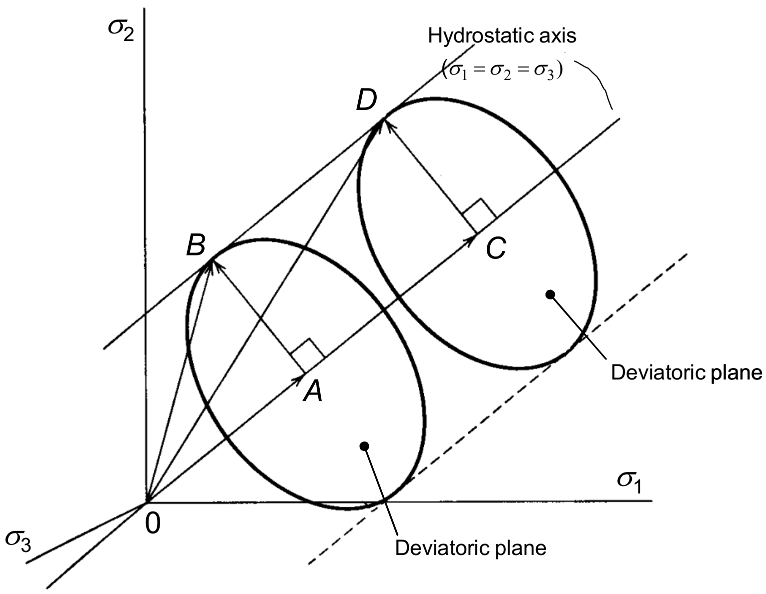
[3 marks]

(c)



[3 marks]

(d)



Hydrostatic stress is given by:

$$\sigma_h = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

And deviatoric stress is given by:

$$\sigma' = (\sigma_1 - \sigma_h, \sigma_2 - \sigma_h, \sigma_3 - \sigma_h)$$

[3 marks for well explained, complete explanation, not just recalling the diagram, this only gets 2]

(e)

Applied torque creates torsional shear stress:

$$\tau = \frac{Tr}{J} = \frac{2T}{\pi r^3} = \frac{2 \times 2000}{\pi \times (20 \times 10^{-3})^3} = 1.59 \times 10^8 \, Pa = \ 159 \text{ MPa}$$

[2 marks]

Bending stress is given by:

$$\sigma = \frac{4M}{\pi r^3}$$

[2 marks]

Tresca, Including SF of 2:

$$\tau_{max} = \frac{\sigma_2 - \sigma_1}{2} = R = 200 \text{ MPa}$$

$$\therefore 200 = \sqrt{\left(\frac{4M^2}{\pi^2 r^6} + 159^2\right)}$$

$$\therefore \boldsymbol{M = 1522 \text{ Nm}}$$

[3 marks]

von Mises

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_y^2 = 400^2$$

$$\therefore (C+R)^2 - (C+R)(C-R) + (C-R)^2 = 400^2$$

$$\therefore C^2 + 3R^2 = 400^2$$

$$\therefore \frac{16M^2}{\pi^2 r^6} + 3 \times 159^2 = 400^2$$

$$\therefore \boldsymbol{M = 1822.7 \text{ Nm}}$$

[3 marks]